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Gravitational lensing of gravitational waves from merging neutron star binaries

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Abstract

We discuss the gravitational lensing of gravitational waves from merging neutron star binaries, in the context of advanced LIGO type gravitational wave detectors. We consider properties of the expected observational data with cut on the signal-to-noise ratio ρ, i.e., $\rho > \rho_0$. An advanced LIGO should see unlensed inspiral events with a redshift distribution with cut-off at a redshift $z_{\rm max} < 1$ for $h \leq 0.8$. Any inspiral events detected at $z > z_{\rm max}$ should be lensed. We compute the expected total number of events which are present due to gravitational lensing and their redshift distribution for an advanced LIGO in a flat Universe. If the matter fraction in compact lenses is close to 10%, an advanced LIGO should see a few strongly lensed events per year with $\rho > 5$.

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I. INTRODUCTION

An advanced LIGO may observe gravitational waves produced as distant close neutron star binary pairs spiral into each other. During the last stage of inspiral the binary emits copious gravitational waves, with increasing frequency as the orbital period decreases, until finally the pair collides and coalesces. LIGO aims to detect the waves emitted during the last 15 minutes of inspiral when the frequency sweeps up from 10 Hz to approximately 10^3 Hz [1].

In this paper, we discuss the gravitational lensing of gravitational waves from merging neutron star binaries, in the context of advanced LIGO type gravitational wave detectors. Following Ref. [2], we consider properties of the expected observational data with cut on the signal-to-noise ratio ρ, i.e., $\rho > \rho_0$. An advanced LIGO should see unlensed events with a redshift distribution with cut-off at a small redshift $z_{\rm max} < 1$ for $h \leq 0.8$ [2,4]. We argue below that there may be a significant number of inspiral events detected at $z > z_{\rm max}$ which can be detected because they are magnified due to gravitational lensing. We compute the expected total number of events which are present due to gravitational lensing and their redshift distribution for an advanced LIGO, for plausible choices of cosmological parameters.

The aforementioned frequency range over which LIGO can detect neutron star binary inspirals corresponds to a wavelength of gravitational waves from 3×10^4 to 10^2 km. This wavelength will be much smaller than the characteristic scales of gravitational fields the gravitational waves are likely to encounter as they pass between the neutron star binaries and the Earth. This means that one may treat the propagation of gravitational waves in the geometrical optics limit [3]. In other words the gravitational lensing magnification will be the same as for optical light and one may use the standard formulae from optical gravitational lens theory.

II. OBSERVATION OF UNLENSED EVENTS

Neutron star binary merger rate at redshift z per unit observer time interval per unit volume is $\dot{n}_m = \dot{n}_0\,(1+z)^2\,\eta(z)$, where $\dot{n}_0$ is the local neutron star binary merger rate per unit volume, $(1+z)^2$ accounts for the shrinking of volumes with redshift (assuming constant comoving volume density of the merger rate) and time dilation, and $\eta(z) = (1+z)^\beta$ describes evolutionary effects. We use the "best guess" local rate density, $\dot{n}_0 \simeq (9.9 + 0.6\,h^2)\,h \times 10^{-8}\mathrm{Mpc}^{-3}\mathrm{yr}^{-1} \simeq 10^{-7}h\,\mathrm{Mpc}^{-3}\mathrm{yr}^{-1}$. [5,6].

In the last stage of a neutron star binary inspiral, gravitational radiation energy losses should lead to highly circular binary orbits. In the Newtonian/quadrupole approximation, for a circular orbit, the rate at which the frequency of the gravitational waves sweeps up or "chirps", is determined solely by the binary's "intrinsic chirp mass", $\mathcal{M}_0 \equiv (M_1 M_2)^{3/5}/(M_1+M_2)^{1/5}$, where M_1 and M_2 are the two bodies' masses. For a binary inspiral source located at redshift z, the detectors measure $\mathcal{M} \equiv \mathcal{M}_0(1+z)$, which is referred to as the *observed* chirp mass. For a given detector, the signal-to-noise ratio is [2]

$$\rho(z) = 8\Theta\,\frac{r_0}{d_L(z)}\left(\frac{\mathcal{M}(z)}{1.2\mathrm{M}_\odot}\right)^{5/6}\,\zeta(f_{\max}), \tag{1}$$

d_L is our luminosity distance to the binary inspiral source. r_0 and $\zeta(f_{\max})$ depend only on the detector's noise power spectrum. The characteristic distance r_0 gives an overall sense of the depth to which the detector can "see". For advanced LIGO, $r_0 = 355\,\mathrm{Mpc}$. $\zeta(f_{\max})$ reflects the overlap of the signal power with the detector bandwidth ($0 \le \zeta \le 1$). For source redshift z, $\zeta \simeq 1$ for $1+z \le 10\,[2.8\mathrm{M}_\odot/(M_1+M_2)]$. $\zeta \simeq 1$ is a good approximation in the context of this paper. Θ is the angular orientation function, it arises from the dependence of ρ on the relative orientation of the source and the detector, $0 \le \Theta \le 4$. Although Θ can not be measured, its probability distribution has been found numerically in Ref. [7], $P_\Theta(\Theta, 0 \le \Theta \le 4) \simeq 5\Theta(4-\Theta)^3/256$, $P_\Theta(\Theta, \Theta > 4) = 0$.

The luminosity distance $d_L(z) = (1+z)^2 d_A(z)$, where $d_A(z)$ is the angular diameter distance. In a flat Universe with a cosmological constant $\Omega_\Lambda = 1 - \Omega_0 \ge 0$, [8] $d_A(z) = cH_0^{-1}(1+z)^{-1}\int_0^z \mathrm{d}w\,\left[\Omega_0\,(1+w)^3 + \Omega_\Lambda\right]^{-1/2}$.

The number rate of binary inspiral events seen by a detector on Earth with signal-to-noise ratio $\rho > \rho_0$ per source redshift interval is [2,4]

$$\frac{\mathrm{d}\dot{N}_{NL}(>\rho_0)}{\mathrm{d}z} = 4\pi\dot{n}_0 \left(cH_0^{-1}\right)^3 \left[\frac{d_A(z)}{cH_0^{-1}}\right]^2 \frac{(1+z)\,\eta(z)}{\sqrt{\Omega_0\,(1+z)^3+\Omega_\Lambda}}\, C_\Theta(x), \tag{2}$$

where $C_\Theta(x) \equiv \int_x^\infty \mathrm{d}\Theta\, P_\Theta(\Theta)$ is the probability that a given detector detects a binary inspiral at redshift z with signal-to-noise ratio greater than ρ_0, it decreases with z and acts as a window function; $C_\Theta(x, 0 \leq x \leq 4) = (1+x)\,(4-x)^4/256$, $C_\Theta(x, x>4) = 0$. x is the minimum angular orientation function

$$x = \frac{4}{h\, A(r_0, \rho_0, \mathcal{M}_0)}\,(1+z)^{7/6} \left[\frac{d_A(z)}{cH_0^{-1}}\right], \tag{3}$$

where we have defined parameter A as in Ref. [4],

$$A \equiv 0.4733 \left(\frac{8}{\rho_0}\right) \left(\frac{r_0}{355\,\mathrm{Mpc}}\right) \left(\frac{\mathcal{M}_0}{1.2\mathrm{M}_\odot}\right)^{5/6}. \tag{4}$$

Note that A absorbs the dependence on detector and source properties ρ_0, r_0, and $\mathcal{M}_0$.

Given the detector threshold in terms of the minimum signal-to-noise ratio ρ_0, the maximum redshift of the source that the detector can "see", $z_{\max}$, is given by Eq.(1) with $\Theta = 4$. For advanced LIGO, $z_{\max} < 1$ for $h \leq 0.8$. [2,4] The redshift distribution given by Eq.(2) terminates at $z = z_{\max}$.

III. OBSERVATION OF LENSED EVENTS

For a source at redshift $z^* > z_{\max}$, we denote its signal-to-noise ratio without lensing by $\rho(z^*)$ [given by Eq.(1)] . The source can not be detected in the absence of lensing. With lensing, its signal-to-noise ratio becomes

$$\rho^*(z^*) = \sqrt{\mu}\,\rho(z^*), \tag{5}$$

where μ is the magnification. The source can be detected if $\rho^*(z^*) > \rho_0$, with ρ_0 denoting the detector threshold.

The probability of a source at redshift z being magnified by a factor greater than μ is $P(>\mu, z) = \tau_L(z)\, y^2(\mu)$, for $\tau_L(z) \ll 1$. $\tau_L(z)$ is the optical depth for gravitational lensing, and $y^2(\mu) \simeq \mu^{-2}$ for $\mu \gg 1$. For point mass lenses, we use

$$y^2(\mu) = \begin{cases} 2\left(\frac{\mu}{\sqrt{\mu^2-1}} - 1\right), & \mu > \frac{3}{\sqrt{5}} \\ 1, & \mu \leq \frac{3}{\sqrt{5}}. \end{cases} \tag{6}$$

The above equation leads to underestimation of $y^2(\mu)$ for μ close to 1, which has negligible effect for our purpose. For a source at redshift $z^* > z_{\max}$ to be detected, we need $\mu > \mu_0$, with

$$\mu_0 \equiv \left[\frac{\rho_0}{\rho(z^*)}\right]^2 = \left[\frac{x(z^*)}{\Theta}\right]^2, \tag{7}$$

where $x(z^*)$ is given by Eq.(3). Note that μ_0 depends on the angular orientation function Θ. The larger Θ, the larger the signal-to-noise ratio without lensing [see Eq.(1)], the smaller the magnification needed to reach the detector threshold ρ_0. Since $\Theta = x$ is the minimum angular orientation function needed for a source to be seen without lensing [see Eq.(3)], only events with $\Theta < x$ need be considered when we count the number of events which are present due to gravitational lensing.

The number rate of binary inspiral events which can be seen due to gravitational lensing by a detector on Earth with signal-to-noise ratio $\rho > \rho_0$ per source redshift interval is

$$\frac{\mathrm{d}\dot{N}_L(>\rho_0)}{\mathrm{d}z^*} = 4\pi \dot{n}_0 \left(cH_0^{-1}\right)^3 \left[\frac{d_A(z^*)}{cH_0^{-1}}\right]^2 \frac{(1+z^*)\,\eta(z^*)\,\tau_L(z^*)}{\sqrt{\Omega_0\,(1+z^*)^3 + \Omega_\Lambda}} \int_0^{x(z^*)} \mathrm{d}\Theta\, P_\Theta(\Theta)\, y^2(\mu_0). \tag{8}$$

Note that $\Theta_{\min} = 0$, because we take the maximum magnification to be infinite, which is a reasonable approximation in the context of this paper.

We consider two types of lensing: 1) "macro-lensing" from the large-scale gravitational field of galaxies, and 2) "micro-lensing" from the smaller scale gravitational field from compact objects such as stars. The optical depth from macrolensing is [9]

$$\tau_L^G(z) = \frac{F}{30}\left[\frac{(1+z)\, d_A(z)}{cH_0^{-1}}\right]^3, \tag{9}$$

where F parametrizes the gravitational lensing effectiveness of galaxies [as singular isothermal spheres]. Denoting the matter fraction in compact lenses as $\Omega_{\rm L}$, the optical depth of microlensing is [10,11]

$$\tau_L^p(z) = \frac{3}{2}\,\frac{\Omega_{\rm L}}{\lambda(z)} \int_0^z {\rm d}w\, \frac{(1+w)^3\ [\lambda(z)-\lambda(w)]\ \lambda(w)}{\sqrt{\Omega_0(1+w)^3+\Omega_\Lambda}}, \tag{10}$$

where the affine distance (in units of cH_0^{-1}) is $\lambda(z) = \int_0^z {\rm d}w\,(1+w)^{-2}\left[\Omega_0(1+w)^3+\Omega_\Lambda\right]^{-1/2}$. For $\Omega_\Lambda = 0$, $\tau_L^p(z) = \frac{3}{5}\,\Omega_{\rm L}\,\left[(y^{5/2}+1)\ln y/(y^{5/2}-1) - \frac{4}{5}\right]$, where $y = 1+z$.

IV. PREDICTIONS/SPECULATION

To make more specific predictions we must choose parameters and to do this we are forced to speculate on the rate of inspiral events and the sensitivity of future gravitational wave detectors. Typically we expect neutron star binaries to have $\mathcal{M}_0 = 1.2\,{\rm M}_\odot$ while the advanced LIGO might have $r_0 = 355\,{\rm Mpc}$, $\rho \geq \rho_0 = 5$ then implies $A = 0.7573$. We parametrize evolutionary effects by $\eta(z) = (1+z)^\beta$, with $\beta \geq 0$ and a redshift cut-off of $z_{\rm stop} = 2.5$. The typical lifetime of a neutron star binary is about 10^8 to 10^9 years [5]; a significant fraction of neutron star binaries formed at $z = 3$ would have merged by $z = 2.5$. Since there seem to be a lot of star formation at $z > 3$ [12], $z_{\rm stop} = 2.5$ is probably reasonable.

We find that the macrolensing rate is largest when there is a sizable cosmological constant, but is still negligible unless there is significant evolution. Even with extreme evolution, e.g. $\Omega_\Lambda = 0.8$, $\Omega_0 = 0.2$, $F = 0.05$ [11] and $\beta = 3$ macrolensing yields only 70% of the number of lensed events of microlensing with more modest parameters: $\Omega_{\rm L} = 0.07$ and $\beta = 0$. Macrolensing of gravitational waves due to galaxies is negligible compared to a plausible microlensing rate. This is partly due to the fact that we have a good idea of the number and properties of galaxies while we are more free to speculate on the number of compact objects and partly due to the fact that point mass lenses are more effective gravitational lenses than galaxies.

For the rest of the paper we restrict ourselves to microlensing, which gives a few strongly lensed events per year without much evolution, for a currently acceptable value of $\Omega_{\rm L}$. We have considered two plausible cosmological models: (1) $\Omega_0=1$ and $\Omega_{\rm L}=0.1$; (2) $\Omega_\Lambda=0.8$, $\Omega_0=0.2$, $\Omega_{\rm L}=0.07$.

We consider expected data with cut on the signal-to-noise ratio, $\rho>5$. Fig.1 shows the expected total number per year of events which are present due to gravitational lensing as function of h, for two cosmological models, with $\beta=0,1$. The solid lines are for $\Omega_0=1$ and $\Omega_{\rm L}=0.1$, and the dashed lines are for $\Omega_\Lambda=0.8$, $\Omega_0=0.2$, and $\Omega_{\rm L}=0.07$. Fig.2 shows the corresponding expected total number per year of events which can be seen without gravitational lensing as function of h, with the same line types as Fig.1. The expected total numbers in both Fig.1 and Fig.2 increase with increasing β, as expected. Fig.3 shows the redshift distribution of expected events corresponding to Figs.1-2 for $h=0.8$. The dotted lines indicate the distribution of expected events which are present due to gravitational lensing. Note that gravitational lensing leads to tails at high redshift. For each cosmological model, the higher tail corresponds to $\beta=1$. Note that in principle, the evolutionary index can be measured from the region of the redshift distribution dominated by events which can be seen without gravitational lensing. Note also that most of the events which are seen due to gravitational lensing lie beyond the cut-off redshift of the events which can be seen without gravitational lensing.

We have used $\mathcal{M}_0=1.2\,M_\odot$ as the typical intrinsic chirp mass of neutron star binary inspirals. It is expected that $\mathcal{M}_0$ will fall in the narrow range of $1.12-1.26\,M_\odot$ [2] while an advanced LIGO can measure the observed chirp mass $\mathcal{M}=(1+z)\mathcal{M}_0$ to an accuracy of better than 0.1% [7,13]. Thus the uncertainty in the redshift of a given event will be very small compared to the large range of z over which the events which are seen due to gravitational lensing are distributed [see Fig.3]. Since the redshift distribution of observed events which can be seen without gravitational lensing should terminate at a relatively small redshift $z_{\rm max}$, an observation of an event with redshift significantly greater than $z_{\rm max}$ is a strong evidence for gravitational lensing. One should be able to identify the events which are seen due to gravitational lensing!

V. DISCUSSION

While most neutron star binary inspiral events detected by an advanced LIGO will probably not be affected by gravitational lensing, there could be a detectable number of events which are significantly magnified via gravitational lensing by compact objects. These lensed events will be easily identifiable by their high observed chirp masses. For the no-evolution parameters used above one would expect around two events per year which are seen due to gravitational lensing. Even a modest evolution ($\beta = 1$) of the rate of inspirals can significantly increase the rate of events which are seen due to gravitational lensing, and one could imagine even stronger evolution. During the lifetime of a detector, say ten years, one might detect dozens of events which are seen due to gravitational lensing, from which one could estimate the amount of matter in compact lenses, $\Omega_{\rm L}$. The absence of such expected lensed events will place an interesting constraint on $\Omega_{\rm L}$.

If we can determine $\Omega_{\rm L}$ in this way, we will have a much better handle on the nature of the dark matter in our Universe. Thus lensing adds utility to the observation of inspiral events, which has already been shown to provide a measure of the Hubble constant, the deceleration parameter, and the cosmological constant [14,15,2,4]. Gravitational lensing will also add additional noise to the determination of these cosmological parameters, although this noise is relatively small [15]. This is because, as we have seen, most inspiral events are little affected by gravitational lensing. Finally we note that our consideration of lensing for inspiral events is much the same as that which one uses when considering supernovae "standard candles" [16,17].

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REFERENCES

[1] K.S. Thorne, in Proceedings of IAU Symposium 165: Compact Stars in Binaries, edited by J. van Paradijs, E. van den Heuvel, and E. Kuulkers (Kluwer Academic Publishers).

[2] L.S. Finn, Phys. Rev. D **53**, 2878 (1996).

[3] C.W. Misner, K.S. Thorne, J.A. Wheeler, "Gravitation" (W.H. Freeman and Company, 1973).

[4] Y. Wang and E.L. Turner, astro-ph/9603034.

[5] E.S. Phinney, Astrophys. J. **380**, L17 (1991).

[6] R. Narayan, T. Piran, and A. Shemi, Astrophys. J. **379**, L17 (1991).

[7] L.S. Finn and D.F. Chernoff, Phys. Rev. D **47**, 2198 (1993).

[8] S. M. Carroll, W. H. Press, and E. L. Turner, Ann. Rev. Astron. Astrophy., **30**, 499 (1992).

[9] E.L. Turner, Astrophys. J. **365**, L43 (1990).

[10] E.L. Turner, J.P. Ostriker, and J.R. Gott, Astrophys. J. **284**, 1 (1984).

[11] Fukugita, M., and Turner, E. L., Mon. Not. Roy. Astr. Soc., **253**, 99 (1991).

[12] C.C. Steidel, M. Giavalisco, M. Pettini, M. Dickinson, and K.L. Adelberger, Astrophys. J. **462**, L17 (1996).

[13] C. Cutler and É. Flanagan, Phys. Rev. D **49**, 2658 (1994).

[14] D.F. Chernoff and L.S. Finn, Astrophys. J. Lett. **411**, L5 (1993).

[15] D. Marković, Phys. Rev. D **48**, 4738 (1993).

[16] Linder, E.V., Schneider, P., Wagoner, R.T. (1988), Astrophys. J. , 324, 786.

[17] J.A. Frieman, Comments in Astrophys., in press (1996).

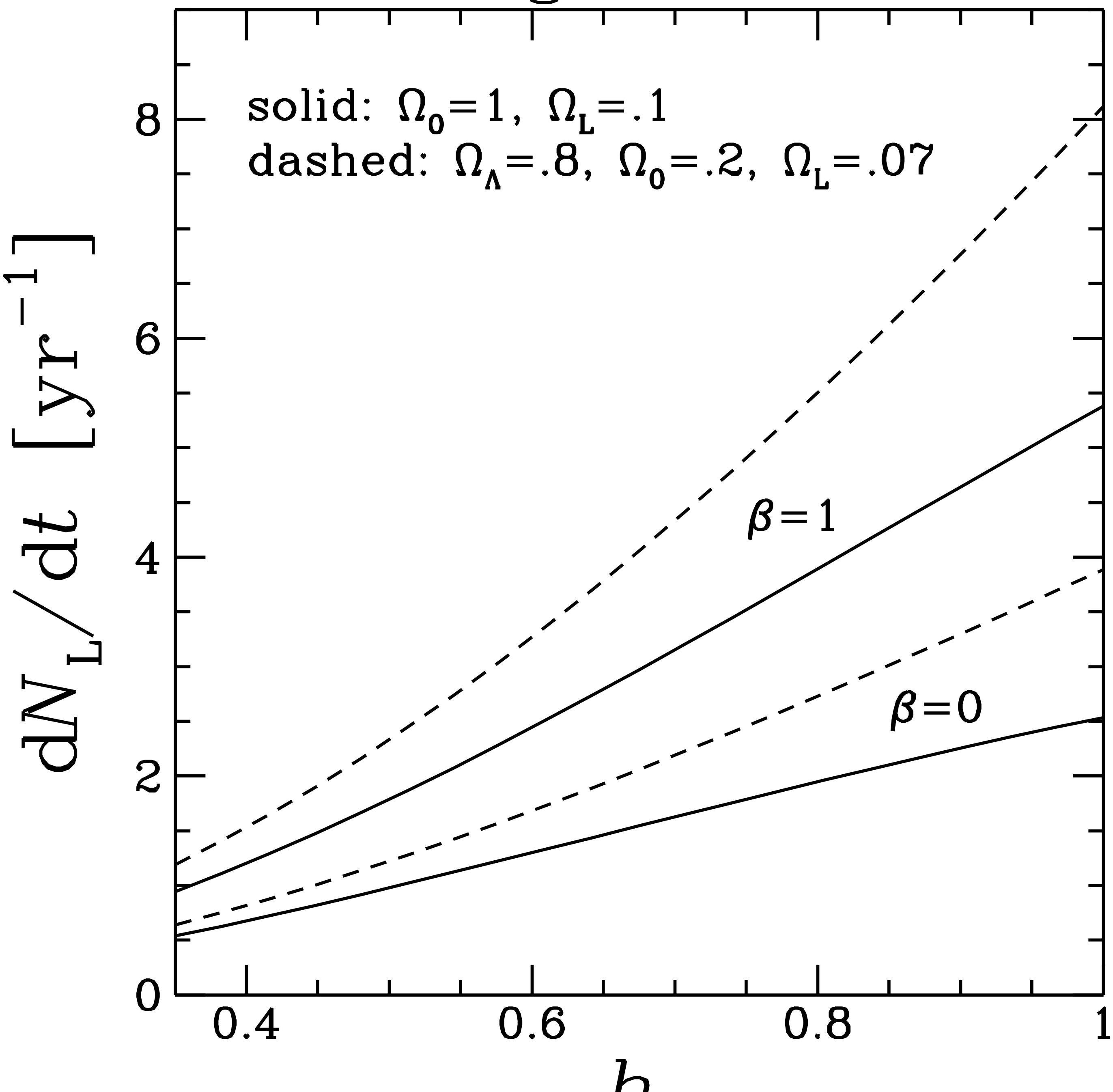
Figure Captions

Fig.1 The total number per year of expected events which are present due to gravitational lensing as function of h, with $\beta = 0, 1$. The solid lines are for $\Omega_0 = 1$ and $\Omega_{\rm L} = 0.1$, and the dashed lines are for $\Omega_\Lambda = 0.8$, $\Omega_0 = 0.2$, and $\Omega_{\rm L} = 0.07$.

Fig.2 The total number per year of expected events which can be seen without gravitational lensing as function of h, with the same line types as Fig.1.

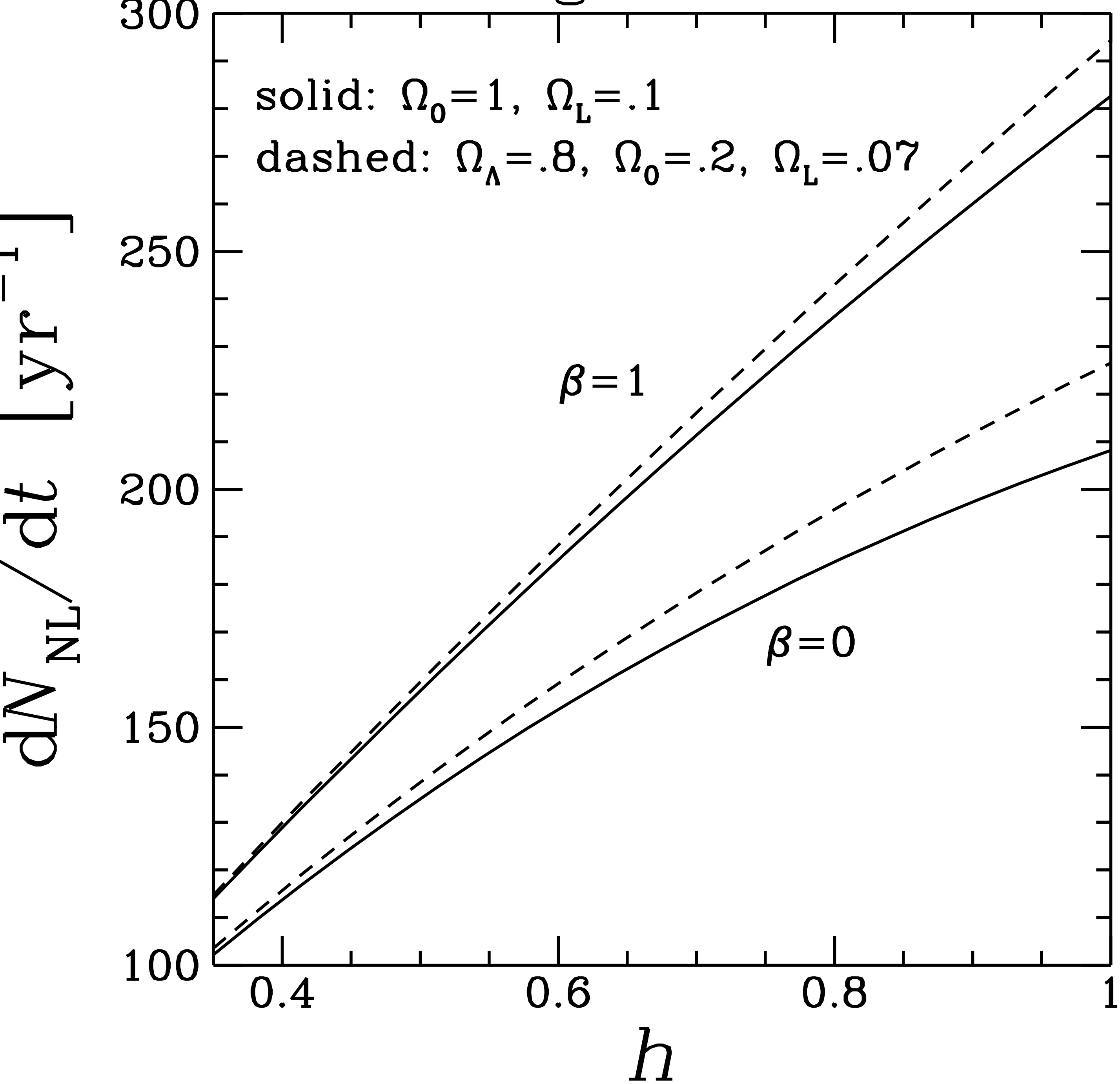
Fig.3 The redshift distribution of expected events corresponding to Fig.1 for $h = 0.8$. The dotted lines indicate the distribution of expected events which are seen due to gravitational lensing. For each cosmological model, the higher tail corresponds to $\beta = 1$.

Figure 1



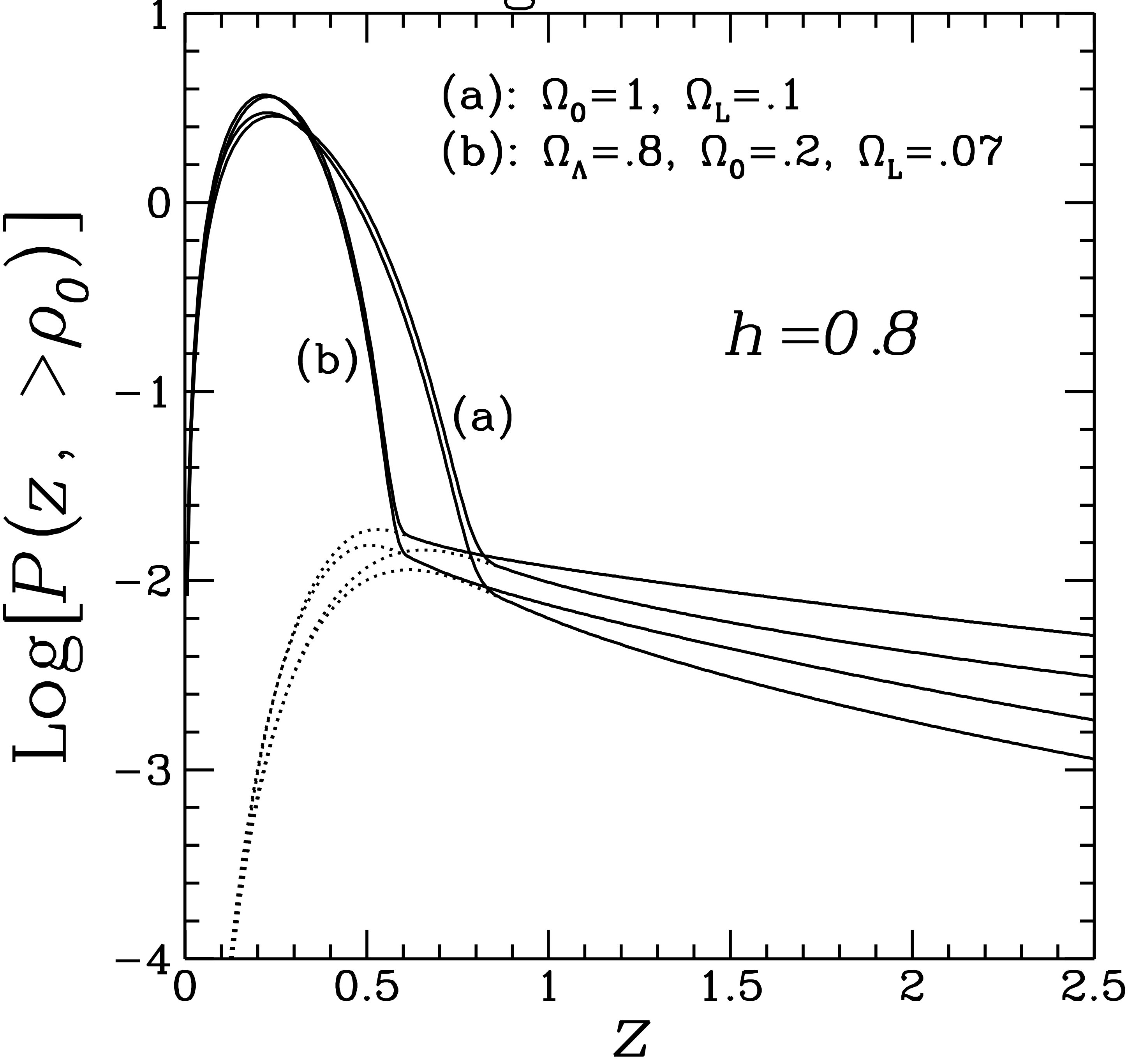
solid: $\Omega_0=1$, $\Omega_L=.1$
dashed: $\Omega_\Lambda=.8$, $\Omega_0=.2$, $\Omega_L=.07$
$\beta=1$
$\beta=0$
dN_L/dt [yr^{-1}]
h
0
2
4
6
8
0.4
0.6
0.8
1

Figure 2



solid: Ω_0=1, Ω_L=.1
dashed: Ω_Λ=.8, Ω_0=.2, Ω_L=.07
$\beta=1$
$\beta=0$
300
250
200
150
100
0.4
0.6
0.8
1
dN_{NL}/dt [yr^{-1}]
h

Figure 3


(a): $\Omega_0=1,\ \Omega_L=.1$
(b): $\Omega_\Lambda=.8,\ \Omega_0=.2,\ \Omega_L=.07$
$h=0.8$
(b)
(a)
$\mathrm{Log}[P(z,\ >\rho_0)]$
z
1
0
−1
−2
−3
−4
0
0.5
1
1.5
2
2.5